Comments on social insurance and the optimum piecewise linear income tax*

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Abstract

Using Varian's social insurance framework with a piecewise linear two bracket income tax, where t_1 is the tax rate in the lower bracket and $t_1 + t_2$ is the tax rate in the upper bracket, Strawczynski (1998) claims that optimality requires $t_1^* < t_2^* = 1$. This note provides three comments: First, it shown that the argument that $t_2^* = 1$ does not necessarily hold. Second, an equally reasonable *interpretation* of the result is that $t_2^* = 0$, if an explicit lump sum tax contingent on luck is allowed. Third, the result also depends crucially on that income differences are solely determined by luck, even if the population is *ex ante* homogenous. In an example it is shown that then $t_2^* = 0$ is optimal.

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Introduction

Strawczynski (1998) employs a two bracket piecewise linear tax schedule to the Varian (1980) framwork of social insurance. In this framework individuals have a fixed and certain income w in period one and save x to period two, where income becomes uncertain. The tax system implies that low income is taxed at the rate t_1 and high income at the rate $t_1 + t_2$. The difference between low and high income is exogenously set so that high income only depends on luck. The information assumption is that luck is verifiable. All tax revenues are redistributed through a uniform lump sum transfer D. In contrast to the simulation results of Slemrod et al. (1994), who analysed the classical optimal tax model, Strawczynski claims that optimal tax rates satisfies $t_1^* < t_2^* = 1$.[1]

I have three comments on this claim: First, Strawczynski's argument that $t_2^* = 1$ does not hold in general. Second, the claimed result depends on an *ad hoc* and only seemingly innocuous restriction of the policy space. If an explicit lump sum tax D_2 contingent on luck is allowed, an assumption consistent with the informational assumptions, then I argue that an equally reasonable interpretation of the result is that $t_2^* = 0$. Third, the result also depends crucially on the formulation of the stochastic mechanism. It will not hold if income differences depend on both luck *and* savings: In an example I show that $0 = t_2^* < t_1^* < 1$ when a lump sum tax D_2 contingent on luck is available in addition to the uniform transfer D.

The sensitivity test performed by Strawczynski (1998) with respect to the piecewise linear tax does therefore not change the overall picture reported by Slemrod et al. (1994), i.e., that marginal tax rates should be decreasing. Instead, what is shown is that a uniform linear tax complemented with a lump sum tax on luck is optimal. I will now first briefly review the model and then in more detail state my comments. A final section contains my conclusions.

[1] A main contribution of Strawczynski, which I will not comment, is to test the sensitivity of Varian's specific assumptions underlying the simulation results. The social insurance framework is different from the classical frawework beacuse savings cannot be made contingent on states of nature. In the classical model, however, individuals of different productivity types are not restricted to choose the same labour supply. The social insurance framework is therefore in a sense more akin to the model used by e.g., Eaton and Rosen (1980) where a representative individual chooses one level of labour supply in the presence of wage rate uncertainty. Sheshinski (1989) reported a higher optimal marginal tax rate in the second bracket for the classical model. Slemrod et al. (1994), however, showed that this claim was false and provided simulations that indicated that decreasing marginal tax rates are optimal.

The model

Strawczynski's formulation of the problem is as follows: In period two the realised income is $x + e_i$, where $(e_1, e_2, e_3, e_4) = (-\epsilon_2, -\epsilon_1, \epsilon_1, \epsilon_2)$ are fixed and equally probable random income shocks, such that $\epsilon_1 < \epsilon_2$ and therefore $E(y) = x$. The crucial information assumption is that if the shock ϵ_2 is realised this can be verified by the policy maker. All income is taxed at the rate t_1 but if the state ϵ_2 is observed the additional income in this state, i.e., $\epsilon_2 - \epsilon_1$, is taxed at an *additional* rate t_2. All tax revenue is redistributed with the uniform lump sum transfer D, where a balanced budget implies $D = t_1 x + 0.25 t_2(\epsilon_2 - \epsilon_1)$. The realised period two income net of taxation therefore is

$$y_i = \begin{cases} (1-t_1)(x+e_i) + D & \text{if } i = 1,2,3 \\ (1-t_1)(x+\epsilon_2) - t_2(\epsilon_2 - \epsilon_1) + D & \text{otherwise.} \end{cases} \tag{1}$$

The expected utility of savings x for a given tax system is $u(w - x) + 0.25 \sum_{i=1}^{4} u(y_i)$, where u is a strictly concave *ex post* utility function. Individuals are assumed to save a strictly positive amount x^* according to the first order condition

$$u'(w - x^*) = \frac{1}{4}(1 - t_1) \sum_{i=1}^{4} u'(y_i^*), \tag{2}$$

where y_i^* is realised period two income net of taxation given x^*. The policy maker chooses the parameters of the tax system so as to maximise the utility of a representative individual, subject to the budget constraint and given that the individual chooses (strictly positive) savings so as to maximise utility for the chosen tax system. Strawcynski then argues that the optimal solution implies $t_1^* < t_2^* = 1$.

Comment 1

The policy maker's problem is

$$\max_{t_1,t_2,D} u(w - x^*) + 0.25 \sum_{i=1}^{4} u(y_i^*) \text{ s.t. } D = t_1 x^* + 0.25 t_2(\epsilon_2 - \epsilon_1). \tag{3}$$

First, note that the first order conditions for an optimal policy (t_1^*, t_2^*)

can, when the budget constraint is used to eliminate D^*, be written as

$$\operatorname{cov}(u', e) = t_1^* \frac{1}{4} \frac{\partial x^*}{\partial t_1} \sum_{i=1}^{4} u'(y_i^*) \text{ and} \tag{4a}$$

$$\frac{1}{4}(\epsilon_2 - \epsilon_1)\upsilon(t_1^*, t_2^*) = t_1^* \frac{\partial x^*}{\partial t_2} \sum_{i=1}^{4} u'(y_i^*), \tag{4b}$$

where $\operatorname{cov}(u', e) := \frac{1}{4}\sum_{i=1}^{4} e_i u'(y_i^*)$ and $\upsilon(t_1^*, t_2^*) := \sum_{i=1}^{3}(u'(y_4^*) - u'(y_i^*))$. Equation (4a) is the analogue to the first order condition for an optimal linear tax in Varian (1980) and Eaton and Rosen (1980). Since $\operatorname{cov}(u', e) < 0$ it implies that $t^* > 0$. Since (2) implies $t_1^* < 1$ we have that $t_1^* \in (0, 1)$.

Strawczynski's argument is that $y_4^* \geq y_3^* > y_2^* > y_1^*$ for all $t_2 \in [0, 1]$. Since y_4^* is decreasing and y_i^* $i = 1, 2, 3$ is increasing in t_2 an increase in the additional second bracket tax rate would then be welfare improving in whole unit interval.

Observe that *the marginal tax rate on period one savings* is t_1 in all states of nature. However, the statuary marginal tax rate on realised income is t_1 in the unlucky states and $t_1 + t_2$ in the lucky state. To illustrate this we can rewrite realised income in the lucky state as

$$y_4^* = (1 - t_1)(x^* + \epsilon_1) + (1 - t_1 - t_2)(\epsilon_2 - \epsilon_1) + D. \tag{5}$$

Then $y_4^* - y_3^* = (1 - t_1 - t_2)(\epsilon_2 - \epsilon_1) \geq 0$ only as long as $t_2 \leq 1 - t_1$, with strict equality if $t_2 = 1 - t_1$. Furthermore, for $t_2 > 1 - t_1$ we have $y_4^* < y_3^*$ and at some point an increase in t_2 would potentially not be welfare improving; i.e., $\upsilon(t_1^*, t_2^*) > 0$. It then remains to be shown that this does not occur for $t_2 \in (1 - t_1, 1)$. As an example, consider the case if $t_1^* > \frac{\epsilon_1 + \epsilon_2}{2\epsilon_2}$ at $t_2^* = 1$. Then $y_4^* < y_i^*$ $\forall i = 1, 2, 3$ and the right hand side of (4b) would be positive and therefore $t_2^* = 1$ would not be optimal.

Hence, with the argument presented so far, the second bracket additional marginal tax rate may or may not equal unity. What can be shown, however, is that $t_1^* + t_2^* > 1$. For $t_2 = 1 - t_1^*$ we have that $y_4^* = y_3^*$ and $\upsilon(t_1^*, 1 - t_1^*) < 0$. Since $\frac{\partial \upsilon}{\partial t_2} > 0$ by $u'' < 0$, the unique additional marginal tax rate in the second bracket must satisfy $t_2^* > 1 - t_1^*$. The total statuary marginal tax rate on additional realised income in the second bracket is then (in a sense) larger than 100 per cent; i.e., $t_1^* + t_2^* > 1$. Therefore, even if we have not shown that $t_1^* < t_2^* = 1$, optimal tax marginal rates seem to be increasing; $t_1^* < 1 < t_1^* + t_2^*$. In this model luck should be taxed away, but luck is taxed at the rate $t_1 + t_2$ not t_2.

Comment 2

I shall now show that even this modified result is not robust for a small enlargement of the policy space that is consistent with the assumption that luck (ϵ_2) is verifiable. We now allow for different lump sum taxes/transfers for the two brackets so that D_2 is the additional lump sum tax paid in the second bracket (i.e., contingent on luck), so that realised income now is

$$y_i^* = \begin{cases} (1-t_1)(x^* + e_i) + D & \text{if } i = 1, 2, 3 \\ (1-t_1)(x^* + \epsilon_2) - t_2(\epsilon_2 - \epsilon_1) + D - D_2 & \text{otherwise.} \end{cases} \tag{6}$$

The policy maker's budget constraint then is

$$D = t_1 x^* + \frac{t_2}{4}(\epsilon_2 - \epsilon_1) + \frac{1}{4} D_2. \tag{7}$$

We focus on the second bracket tax parameters given that $t_1^* \in (0, 1)$, which can be shown to hold. The planner's first order conditions for t_2^* and D_2^* are

$$\frac{1}{(\epsilon_2 - \epsilon_1)} \frac{\partial x^*}{\partial t_2} = \frac{1}{4\lambda^* t_1^*} \left[u'(y_4^*) - \lambda^* \right] \text{ and} \tag{8a}$$

$$\frac{\partial x^*}{\partial D_2} = \frac{1}{4\lambda^* t_1^*} \left[u'(y_4^*) - \lambda^* \right], \tag{8b}$$

where λ^* is the shadow price of additional tax revenues in the optimal solution. Since the individual first order condition implies $\frac{\partial x^*}{\partial t_2} = (\epsilon_2 - \epsilon_1) \frac{\partial x^*}{\partial D_2}$ equations (8a) and (8b) are linearly dependent. There will therefore be infinitly many solutions (t_2^*, D_2^*), two of which are $(t_2^* = 0, D_2^* > 0)$ and $(t_2^* > 0, D_2^* = 0)$. The optimal tax in the second bracket in Strawczynski (1998) works as a lump sum tax and can therefore, of course, be replaced by an explicit lump sum tax. Such a lump sum tax is consistent with the informational structure.

A more reasonable interpretation of the result is, in my opinion, therefore to say that the *additional* second marginal tax rate is equal to zero: The decision that the individual is making is what amount to save to period 2 and t_2 only has a pure income effect on that decision.

Comment 3

Finally, I will show that the results derived in the previous section are not robust to a small change in the income generating process. I now assume that income differences do not only depend on luck but also on individual behaviour. Given the tax system of the previous section the result will be that $t_2^* = 0$.

Therefore, let realised income in period two be xe_i, where e_i $i = 1, 2, 3, 4$ are random shocks such that $e_i < e_{i+1}$, the expectation of which is assumed to equal unity. The four states of nature are still equally probable. The tax system is the same as in the previous section but now the additional income in the 'lucky' state is $x(e_4 - e_3)$.

If we in a standard way combine the first order conditions for (t_1^*, D^*) and (t_2^*, D_2^*) we get

$$\operatorname{cov}(u', x^*e) = \lambda^* \left(t_1^* + \frac{t_2^*}{4}(e_4 - e_3) \right) \left[\frac{\partial x^*}{\partial t_1} + x^* \frac{\partial x^*}{\partial D} \right] \text{ and} \tag{9a}$$

$$0 = \left(t_1^* + \frac{t_2^*}{4}(e_4 - e_3) \right) \left[\frac{\partial x^*}{\partial t_2} - x^*(e_4 - e_3) \frac{\partial x^*}{\partial D_2} \right]. \tag{9b}$$

Note that the expressions within brackets are the relevant substitutions effects which are both strictly negative. I now will show that $t_2^* \neq 0$ cannot be part of the optimal solution. Suppose first that t_1^* and t_2^* are both strictly positive (the case where both rates are strictly negative is clearly not optimal by (9a)). This is consistent with (9a) but contradicted by (9b). Equation (9b) seems to imply that $t_1^* + \frac{t_2^*}{4}(e_4 - e_3) = 0$ so that the tax rates have different non–zero signs, but that is on the other hand contradicted by (9a). Therefore, t_1^* and t_2^* cannot both be strictly positive. If we on the other hand exclude the second bracket additional marginal tax rate the first order condition for D_2^* is

$$\frac{1}{4}(u'(y_4^*) - \lambda^*) = \lambda^* t_1^* \frac{\partial x^*}{\partial D_2}, \tag{10}$$

which together with (9a) evaluated at $t_2^* = 0$ do not generate a similar inconsistency. Therefore, the use of an additional marginal tax rate is not part of an optimal solution. Still $t_1^* \in (0, 1)$.

Conclusion

In my opinion Strawczynski's contribution does not contradict the picture painted by earlier contributions. However, I do not argue that the optimal piecewise linear tax system implies decreasing marginal tax rates in either the classical optimal tax framework or in the social insurance framework. The results presented here for the insurance model indicate that a linear tax in combination with a lump sum tax contingent on luck is is optimal. But this conclusion depends crucially on the information assumption; i.e., that luck is verifiable.

Other differences between the classical and the insurance framework that may affect the results are that (i) the cut–off income defining the two brackets in the present discussion has been assumed to be fixed at an arbitrary income so that the second bracket only contains one state of nature and (ii) the additional problem that high performing agents may imitate low performing agents is not present in the insurance framework.[2] Determining the cut–off income endogenously may be of importance, in particular if luck is not possible to verify.

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[2]In a simulation Strawczynski considers a heterogenous population model, where the government's decision problem is described in Appendix A. However, there is no reference to the imitation problem, whether it is trivial or not. In the traditional two type optimal tax model with piecewise linear schedules, however, Lundholm (1991) shows that the traditional non–linear result holds; i.e., $0 = t_1^* + t_2^* < t_1^* < 1$.